

Which ones among our cities are resilient? Comparison of ranking results of methods of linear ordering

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Resilience of a city

Ability of a city to return to a stable path of development after crisis, its dangers and consequences

Introduction of mechanisms allowing to quickly and efficiently cope with threats, abilities to adjust and rebuilt itself

Aim

- To identify most and least resilient voivodeship capitals in Poland in 2021
- To compare results of different methods of linear ordering to determine impact of a method on ranking results

Data set

- 18 cities: Poland's voivodeship capitals
- 20 variables representing different aspects of city resilience in 2021



Number	Variable	Area of resilience
1	Year on year change in own revenue derived from a share in revenue from corporate income taxes per capita (%)	Economic resilience
2	Number of small and medium enterprises per capita	
3	Average monthly gross earnings (PLN)	
4	Year on year change in population (%)	Social resilience
5	Year on year change in number of working persons per capita (%)	
6	Year on year change in number of unemployed per capita (%)	
7	Net primary education enrolment rate (%)	
8	Number of university graduates per 1000 inhabitants	
9	Number of women working per number of women of working age (15-59 years olds)	
10	Number of cars per 1000 inhabitants	
11	Number of foundations, associations and social organizations per 1000 inhabitants	
12	Proportion of investment expenditure in total expenditure (%)	Institutional resilience
13	Funds from European Union on financing Union programs and projects (PLN) per 1000 inhabitants	
14	Expenditure on public debt servicing (PLN) per 1000 inhabitants	
15	Current expenditure on public safety and fire protection (PLN) per 1000 inhabitants	
16	Length of district and communal hard-surfaced roads (kms per 100 km ²)	Infrastructural resilience
17	Number of dwellings per 1000 inhabitants	
18	Number of doctors per 10 000 inhabitants	
19	Number of hospital beds per 1000 inhabitants	
20	Number of primary and general education high schools per 1000 inhabitants	

Hellwig's Measure of Economic Development

1. Standardize all variables $z_{ij} = \frac{x_{ij} - \bar{x}_j}{s_j}$
2. Define pattern object $z_0 = [z_{01}, z_{02}, \dots, z_{0p}]$
3. Calculate distance from each object to the pattern

$$d_{i0} = \sqrt{\sum (z_{ij} - z_{0j})^2}$$

4. Calculate measure of development and rank objects

$$m_i = 1 - \frac{d_{i0}}{d_0}$$

where $d_0 = \bar{d} + 2s_d$, \bar{d} and s_d are the mean and standard deviation of d_{i0}

TOPSIS method

1. Standardize all variables
$$z_{ij} = \frac{x_{ij}}{\sqrt{\sum x_{ij}^2}}$$
2. Define pattern and anti-pattern objects z_0 and z_{-0}
3. Calculate distance from each object to the pattern and anti-pattern

$$d_{i0+} = \sqrt{\sum (z_{ij} - z_{0j})^2} \qquad d_{i0-} = \sqrt{\sum (z_{ij} - z_{-0j})^2}$$

4. Calculate measure of development and rank objects

$$q_i = \frac{d_{i0-}}{d_{i0-} + d_{i0+}}$$

TOPSIS method (2)

1. Standardize all variables
$$z_{ij} = \frac{x_{ij} - \bar{x}_j}{s_j}$$
2. Define pattern and anti-pattern objects z_0 and z_{-0}
3. Calculate distance from each object to the pattern and anti-pattern

$$d_{i0+} = \sqrt{\sum (z_{ij} - z_{0j})^2} \qquad d_{i0-} = \sqrt{\sum (z_{ij} - z_{-0j})^2}$$

4. Calculate measure of development and rank objects

$$q_i = \frac{d_{i0-}}{d_{i0-} + d_{i0+}}$$

Method of standardized sums

1. Standardize all variables

$$z_{ij} = \frac{x_{ij} - \bar{x}_j}{s_j} \quad \text{for stimulants}$$

$$z_{ij} = \frac{\bar{x}_j - x_{ij}}{s_j} \quad \text{for dis-stimulants}$$

2. Calculate average of standardised values

$$w_i = \frac{1}{p} \sum z_{ij} \quad \text{where } p = \text{number of variables}$$

Method of ranks

1. Assign rank r_{ij} to each object i according to each variable j
 - From the highest (best) to lowest (worst) for stimulants
 - From the lowest (best) to the highest (worst) for dis-stimulants
2. Calculate average of ranks

$$v_i = \frac{1}{p} \sum r_{ij} \quad \text{where } p = \text{number of variables}$$

Ranking results in 2021

Rank	City	Hellwig
1	Poznań	0,475
2	Kraków	0,383
3	Opole	0,366
4	Warszawa	0,316
5	Katowice	0,310
6	Rzeszów	0,305
7	Wrocław	0,279
8	Lublin	0,273
9	Olsztyn	0,209
10	Gdańsk	0,194
11	Kielce	0,191
12	Łódź	0,155
13	Szczecin	0,154
14	Białystok	0,129
15	Toruń	0,125
16	Zielona Góra	0,084
17	Bydgoszcz	0,080
18	Gorzów Wielkopolski	0,065

Rank	City	TOPSIS
1	Rzeszów	0,582
2	Zielona Góra	0,552
3	Opole	0,543
4	Kraków	0,521
5	Poznań	0,508
6	Gdańsk	0,490
7	Wrocław	0,489
8	Lublin	0,465
9	Warszawa	0,459
10	Olsztyn	0,444
11	Białystok	0,435
12	Bydgoszcz	0,421
13	Toruń	0,415
14	Szczecin	0,406
15	Gorzów Wielkopolski	0,400
16	Katowice	0,397
17	Kielce	0,359
18	Łódź	0,333

Rank	City	TOPSIS 2
1	Poznań	0,615
2	Warszawa	0,558
3	Kraków	0,541
4	Katowice	0,540
5	Opole	0,534
6	Rzeszów	0,521
7	Lublin	0,491
8	Wrocław	0,475
9	Olsztyn	0,449
10	Gdańsk	0,421
11	Białystok	0,411
12	Kielce	0,411
13	Zielona Góra	0,389
14	Szczecin	0,380
15	Gorzów Wielkopolski	0,372
16	Łódź	0,372
17	Toruń	0,365
18	Bydgoszcz	0,360

Ranking results in 2021

Rank	City	Hellwig
1	Poznań	0,475
2	Kraków	0,383
3	Opole	0,366
4	Warszawa	0,316
5	Katowice	0,310
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9	Olsztyn	0,209
10	Gdańsk	0,194
11	Kielce	0,191
12	Łódź	0,155
13	Szczecin	0,154
14	Białystok	0,129
15	Toruń	0,125
16	Zielona Góra	0,084
17	Bydgoszcz	0,080
18	Gorzów Wielkopolski	0,065

Rank	City	Sums
1	Poznań	0,741
2	Warszawa	0,517
3	Katowice	0,466
4	Kraków	0,386
5	Opole	0,358
6	Rzeszów	0,338
7	Lublin	0,191
8	Wrocław	0,076
9	Olsztyn	0,018
10	Gdańsk	-0,163
11	Kielce	-0,175
12	Białystok	-0,273
13	Łódź	-0,330
14	Szczecin	-0,342
15	Zielona Góra	-0,435
16	Gorzów Wielkopolski	-0,445
17	Toruń	-0,445
18	Bydgoszcz	-0,484

Rank	City	Ranks
1	Poznań	5,450
2	Kraków	6,800
3	Katowice	7,450
4	Opole	7,600
5	Warszawa	7,700
5	Rzeszów	7,700
7	Lublin	8,300
8	Wrocław	8,800
9	Olsztyn	9,550
10	Gdańsk	10,200
11	Białystok	10,650
12	Kielce	10,950
13	Łódź	11,000
14	Szczecin	11,450
15	Gorzów Wielkopolski	11,550
16	Toruń	11,650
17	Zielona Góra	12,100
17	Bydgoszcz	12,100

Ranking comparison

1. Pearson's correlation coefficient

	Hellwig	TOPSIS	TOPSIS 2	Sums	Ranks
Hellwig		0,475	0,960	0,964	-0,976
TOPSIS			0,526	0,457	-0,488
TOPSIS 2				0,993	-0,983
Sums					-0,984
Ranks					

2. Spearman's correlation coefficient

	Hellwig	TOPSIS	TOPSIS 2	Sums	Ranks
Hellwig		0,459	0,936	0,965	0,967
TOPSIS			0,525	0,416	0,428
TOPSIS 2				0,981	0,959
Sums					0,975
Ranks					

Ranking results in 2021

City	Hellwig	TOPSIS	TOPSIS 2	Sums	Ranks
Wrocław	7	7	8	8	8
Bydgoszcz	17	12	18	18	17
Toruń	15	13	17	17	16
Lublin	8	8	7	7	7
Gorzów Wielkopolski	18	15	15	16	15
Zielona Góra	16	2	13	15	17
Łódź	12	18	16	13	13
Kraków	2	4	3	4	2
Warszawa	4	9	2	2	5
Opole	3	3	5	5	4
Rzeszów	6	1	6	6	5
Białystok	14	11	11	12	11
Gdańsk	10	6	10	10	10
Katowice	5	16	4	3	3
Kielce	11	17	12	11	12
Olsztyn	9	10	9	9	9
Poznań	1	5	1	1	1
Szczecin	13	14	14	14	14

Ranking results - Poznań

Area	Best results (top 3 places):	Worst results (last 3 places):
Economic	Number of small and medium enterprises	
Social	School enrolment rate, Number of graduates, Number of foundations, associations and social organizations	
Institutional	Proportion of investment expenditure	<i>Funds from European Union (13th place)</i>
Infrastructure	Number of schools	

Ranking results – Warszawa

Area	Best results (top 3 places):	Worst results (last 3 places):
Economic	Number of small and medium enterprises, Average gross earnings	Change in revenue from CIT
Social	Change in number of working persons, Proportion of women employed, Number of cars, Number of foundations, associations and social organizations	Change in number of unemployed, School enrolment rate
Institutional		Proportion of investment expenditure
Infrastructure	Length of roads, Number of dwellings	

Ranking results - Bydgoszcz

Area	Best results (top 3 places)	Worst results (last 3 places)
Economic		Number of small and medium enterprises
Social	Change in number of unemployed	Change in population, School enrolment rate, Number of graduates, Number of foundations, associations and social organizations
Institutional	<i>Funds from European Union (4th)</i>	Expenditure on public safety
Infrastructure		Number of dwellings, Number of schools

Ranking results – Gorzów Wielkopolski

Area	Best results (top 3 places)	Worst results (last 3 places)
Economic		Average earnings
Social	Change in number of unemployed	Change in population, Number of graduates, Number of foundations, associations and social organizations,
Institutional	Proportion of investment expenditure, Expenditure on public debt	
Infrastructure		Length of roads, Number of doctors

Ranking results (Hellwig's method)

City	2014	2015	2016	2017	2018	2019	2020	2021
Wrocław	5	5	6	7	8	6	8	7
Bydgoszcz	16	16	18	18	17	17	16	17
Toruń	10	14	15	15	13	14	12	15
Lublin	7	10	9	9	6	8	7	8
Gorzów Wielkopolski	18	17	17	17	16	18	17	18
Zielona Góra	14	18	16	16	18	16	18	16
Łódź	13	12	11	14	14	15	15	12
Kraków	3	7	2	5	2	3	4	2
Warszawa	2	2	4	3	5	4	3	4
Opole	6	4	3	4	1	7	5	3
Rzeszów	9	8	7	2	7	5	6	6
Białystok	15	15	12	11	12	12	14	14
Gdańsk	11	9	8	8	10	10	13	10
Katowice	4	3	5	6	3	1	2	5
Kielce	12	11	10	10	11	11	9	11
Olsztyn	8	6	13	12	9	9	10	9
Poznań	1	1	1	1	4	2	1	1
Szczecin	17	13	14	13	15	13	11	13

Thank you for your attention